giving therefore the numbers 5, 6, 7, and 8 the relative values thus found, and averaging in the ordinary way, we will find that 6 is the average number of good melons, and therefore $\frac{6}{12}$ or $\frac{1}{2}$ is the probability of selecting a good one.

15. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, in New Windsor College, New Windsor, Maryland.

Todhunter proposes: "From a point in the circumference of a circular field a projectile is thrown at random with a given velocity, which is such that the diameter of the field is equal to the greatest range of the projectile: prove the chance of its falling within the field, is $C=2^{-1}-2\pi^{-1}(\sqrt{2}-1), =.236+$." Is this result perfectly correct as to fact?

Comment on the Solution of Problem 15, by JOHN DOLMAN Jr., Philadelphia, Pennsylvania.

I should not presume to criticise the work of so able and celebrated a mathematician as Professor Matz, did I not consider his solutions of Probability No. 15 vicious in their effects upon the minds of students, striking as they do at the root, not only of the doctrine of mean value and probability, but of the integral calculus itself. "Since the projectiles are *thrown* at random they should *fall* at random," is on a par with, "as an arc varies uniformly the sine varies uniformly," or "because acceleration is constant velocity is constant." The third solution is not clear, but how far any given range the favorable chances can be represented by an area, when the projectiles must fall on the arc of a circle, is difficult to understand. Also in the fifth solution it is stated," for any range PD', the projectiles falling on the circular arc DMD' are within the field" and then the angle PAD' is adopted as uniformly varying, without giving any reason for it. However interesting this may be as mathematical legerdemain, its effects are vicious when it is published without proper explanation for in my humble opinion it is more important that your readers learn to reason correctly than that they be taught to integrate ingeniously.

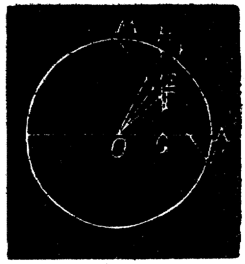
NOTE.—The solution of problem 14 was published without comment for the reason that we considered the solution to be correct, and we confess that we do not yet see the force of Mr. Dolman's argument, though we have not had time to give it much thought.

As to the solutions of problem 15, we hold that the first solution is the only correct solution as that one and that one alone involves the strict literal statement of the problem. It is evident that the number of ways the projectile can be thrown is equal to the surface of a hemisphere whose radius is R. If now we find the surface of that part of this hemisphere any point at which if a projectile be thrown the projectile will fall upon the circular field, diameter R, and then divide this surface by the surface of the hemisphere, the result will be the probability required. This method of solution would have to be accepted by the most critical mind. Professor Matz's first solution involves this principle.—EDITOR.

19. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Find the average area of the circle which is the locus of the middle points of all chords passing through a point taken at random in the surface of a given circle.

I. Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.



Let P be the random point. Through P draw HG perpendicular to OA. Let $OA=a$, $GP=x$, $\angle HOA=\theta$, area circle required$=\frac{\pi}{4}OP^2=\frac{\pi}{4}(x^2+a^2\cos^2\theta)$. An element of the circle at P, is $a\sin\theta d\theta dx$. The limits of θ are 0 and 2π; of x, 0 and $a\sin\theta=x'$. $\triangle=$average area.

$$\therefore\ \triangle=\frac{\int_0^{2\pi}\int_0^{x'}\frac{\pi}{4}(x^2+a^2\cos^2\theta)a\sin\theta d\theta dx}{\int_0^{2\pi}\int_0^{x'}a\sin\theta d\theta dx}$$

$$=\frac{1}{4a}\int_0^{2\pi}\int_0^{x'}(x^2+a^2\cos^2\theta)\sin\theta d\theta dx=\frac{a^2}{12}\int_0^{2\pi}(\sin^2\theta+3\cos^2\theta)\sin^2\theta d\theta=\frac{\pi a^2}{8}.$$

II. **Solution by the PROPOSER.**

Let $OP=x$; then the average area of the circle whose diameter is OP,

$$\text{becomes } A=\int_0^{2\pi}\int_0^{r}(\tfrac{1}{4}\pi x^2)xd\theta dx\div\int_0^{2\pi}\int_0^{r}xd\theta dx=\tfrac{1}{8}\pi r^2.$$

Professor Zerr furnished five different solutions of this problem; Professor Matz seven and Mr. Dolman three.

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

15. **Proposed by SAMUER HART WRIGHT, M.D., M.A.,Ph.D., Penn Yan, Yates Co., New York.**

Required the illuminated area of the Moon's disc when $\frac{2}{3}$ through its first quarter of 60° of longitude east of the Sun, the Earth and Moon being at their mean distances.

II. **Solution by G. B. M. ZERR, A. M., Principal of High School, Staunton. Virginia, and the PROPOSER.**

A rigorous solution of this problem would present many difficulties. In the first place, the surface of the Moon is more than half illuminated; in the second, no observer sees half of the Moon's surface at one time, in the third, the mean distance of the Sun is known within a quarter million miles.

In the following solution we will assume half the Moon's surface illuminated, and half of its surface as presented to an observer at one time. Also we will take the Sun's parallax 8.″81 and hence, his mean distance